

TURBULENT NATURAL CONVECTION IN A POROUS CYLINDRICAL ANNULUS WITH DOUBLE-DIFFUSION EFFECTS

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Abstract. *This paper presents results for coupled heat and mass transport under turbulent flow regime in a horizontal cylindrical annulus filled with a fluid saturated porous medium. Two driving mechanisms are considered to contribute to the overall momentum transport, namely temperature driven and concentration driven mass fluxes. Aiding and opposing flows are considered, where temperature and concentration gradients are either in the same direction or of different sign, respectively. Modeled equations are presented based on the double-decomposition concept, which considers both time fluctuations and spatial deviations about mean values.*

Keywords: *Double-Diffusion, Porous Media, Natural Convection, Turbulence*

1. INTRODUCTION

The study of double-diffusive natural convection in porous media has many environmental and industrial applications, including grain storage and drying, petrochemical processes, oil and gas extraction, contaminant dispersion in underground water reservoirs, electrochemical processes, etc. The importance of double diffusive natural convection can be better appreciated by the volume of papers published in this field, which was reviewed by Nield and Bejan (1999). The analyses of natural convection in a horizontal cylindrical annuli filled by a porous material has been subject of a number of studies in recent years. Thermal insulators, cryogenics, thermal storage systems, electronic cooling, inert gas insulation of high-voltage electric cables and the determination of the requirements for aircraft cabin insulation.

Accordingly, double diffusive convection in a vertical cavity subject to horizontal temperature gradients has been investigated by Trevisan and Bejan (1985, 1986), Goyeau et al. (1996), Mamou et al. (1995, 1998), Mohamad and Bennacer (2002), Nithiarasu et al. (1997), Bennacer et al. (2001, 2003), among others. In most of the aforementioned papers, the intra-pore flow was assumed to be laminar and it was demonstrated that, depending on the governing parameters of the problem and on the thermal to solute buoyancy ratio, various modes of convection prevail.

The natural convection in cylindrical annular geometry filled with porous material also have been studied by distinct numerical approaches, such as the finite-difference method reported by Caltagirone (1976) and Burns and Tien (1979). Finite element method is found in the work of Motjabi et al. (1987).

Motivated by the foregoing, in an earlier paper de Lemos and Tofaneli (2004) a mathematical framework for treating turbulent double-diffusive flows in porous media was presented, but no numerical simulations were published. That work was derived from a general mathematical model for turbulent flow in porous media Pedras and de Lemos (2003), which was developed under a concept called "double-decomposition" de Lemos (2005). Such concept considered time fluctuations of the flow properties in addition to spatial deviations, in relation to a volume-average, when setting up macroscopic equations for the flow. Using such concept, non-buoyant Rocamora and de Lemos 2000 as well as buoyant heat transfer has been considered Braga and de Lemos (2004, 2005, 2006 and 2009) in addition to turbulent mass transfer de Lemos and Mesquita (2003). However, in none of the above applications, results for turbulent double diffusion in porous media were presented.

The purpose of this contribution is to show numerical results for turbulent double-diffusive in porous media, which are obtained with the mathematical model earlier proposed in de Lemos and Tofaneli (2004). To the best of the authors' knowledge, no solutions for turbulent flow using the work in de Lemos and Tofaneli (2004) have been previously published. Here, double-diffusive turbulent natural convection flow in porous media is considered.

2. MATHEMATICAL MODEL

The problem considered here is showed schematically in Fig. 1a and refers to a concentric annulus completely filled with porous material with outer and inner radii r_0 and r_i , respectively, and $R = r_0/r_i = 2$. The top and bottom walls are kept insulated and the porous medium is considered to be rigid. The binary fluid in the cavity of Fig. 1a is assumed to be Newtonian and to satisfy the Boussinesq approximation.

2.1. Macroscopic equations double-diffusion effects

The equations used herein are derived in details in de Pedras and de Lemos (2003), de Lemos and Tofaneli (2004) and de Lemos (2005) and for that their derivation need not be repeated here. They are developed based on volume-averaging procedures which are fully described in the literature Hsu and Cheng (1990), Bear (1972) and Whitaker (1966, 1967).

The macroscopic continuity equation is then given by,

$$\nabla \bar{\mathbf{u}}_D = 0 \quad (1)$$

where the Dupuit-Forchheimer relationship, $\bar{\mathbf{u}}_D = \phi \langle \bar{\mathbf{u}} \rangle^i$, has been used and $\langle \bar{\mathbf{u}} \rangle^i$ identifies the intrinsic (liquid) average of the local velocity vector $\bar{\mathbf{u}}$. The macroscopic time-mean Reynolds equation for an incompressible fluid with constant properties is given as,

$$\rho \nabla \cdot \left(\frac{\bar{\mathbf{u}}_D \bar{\mathbf{u}}_D}{\phi} \right) = -\nabla (\phi \langle \bar{p} \rangle^i) + \mu \nabla^2 \bar{\mathbf{u}}_D + \nabla \cdot \left(-\rho \phi \langle \bar{\mathbf{u}}' \bar{\mathbf{u}}' \rangle^i \right) - \rho \mathbf{g} \phi \left[\beta_\phi (\langle \bar{T} \rangle^i - T_{ref}) + \beta_{C_\phi} (\langle \bar{C} \rangle^i - C_{ref}) \right] - \left[\frac{\mu \phi}{K} \bar{\mathbf{u}}_D + \frac{c_F \phi \rho}{\sqrt{K}} |\bar{\mathbf{u}}_D| \bar{\mathbf{u}}_D \right] \quad (2)$$

where the last two terms in Eq. (2) represent the Darcy-Forchheimer contribution. The symbol K is the porous medium permeability, c_F is the form drag coefficient (Forchheimer coefficient), $\langle \bar{p} \rangle^i$ is the intrinsic average pressure of the fluid, ρ is the fluid density, μ represents the fluid viscosity and ϕ is the porosity of the porous medium. Buoyancy effects due to temperature and concentration variations within the cavity are also shown in Eq. (2). The macroscopic Reynolds stress $-\rho \phi \langle \bar{\mathbf{u}}' \bar{\mathbf{u}}' \rangle^i$ is modeled as,

$$-\rho \phi \langle \bar{\mathbf{u}}' \bar{\mathbf{u}}' \rangle^i = \mu_{i_\phi} 2 \langle \bar{\mathbf{D}} \rangle^v - \frac{2}{3} \phi \rho \langle k \rangle^i \mathbf{I} \quad (3)$$

where

$$\langle \bar{\mathbf{D}} \rangle^v = \frac{1}{2} \left[\nabla (\phi \langle \bar{\mathbf{u}} \rangle^i) + [\nabla (\phi \langle \bar{\mathbf{u}} \rangle^i)]^T \right] \quad (4)$$

is the macroscopic deformation tensor, $\langle k \rangle^i = \langle \bar{\mathbf{u}}' \cdot \bar{\mathbf{u}}' \rangle^i / 2$ is the intrinsic turbulent kinetic energy, k and μ_{i_ϕ} , is the turbulent viscosity, which is modeled in de Lemos (2005) similarly to the case of clear flow, in the form,

$$\mu_{i_\phi} = \rho c_\mu \frac{\langle k \rangle^i}{\langle \varepsilon \rangle^i} \quad (5)$$

Coefficients β_ϕ and β_{C_ϕ} in Eq. (2) are used to write the Grashof numbers associated with the thermal and solute drives, in the form,

$$Gr_\phi = \frac{g \beta_\phi \Delta T H^3}{\nu^2}, \quad Gr_{C_\phi} = \frac{g \beta_{C_\phi} \Delta C H^3}{\nu^2} \quad (6)$$

where $\Delta T = T_1 - T_2$ and $\Delta C = C_1 - C_2$ are the maximum temperature and concentration variation across the cavity, respectively. One should note that for opposing thermal and concentrations drives, such maximum differences are of opposing signs.

The ratio of Grashof numbers defines the buoyancy ratio, N , in the form

$$N = \frac{Gr_{C_\phi}}{Gr_\phi} = \frac{\beta_{C_\phi} \Delta C}{\beta_\phi \Delta T} \quad (7)$$

giving for Eq. (2),

$$\rho \nabla \cdot \left(\frac{\bar{\mathbf{u}}_D \bar{\mathbf{u}}_D}{\phi} \right) = -\nabla \cdot (\phi \langle \bar{p} \rangle^i) + \mu \nabla^2 \bar{\mathbf{u}}_D + \nabla \cdot \left(-\rho \phi \langle \bar{\mathbf{u}} \bar{\mathbf{u}} \rangle^i \right) - \rho \mathbf{g} \phi \beta_\phi \left\{ (\langle \bar{T} \rangle^i - \bar{T}_{ref}) + N \frac{\Delta C}{\Delta T} (\langle \bar{C} \rangle^i - \bar{C}_{ref}) \right\} - \left[\frac{\mu \phi}{K} \bar{\mathbf{u}}_D + \frac{c_F \phi \rho |\bar{\mathbf{u}}_D| \bar{\mathbf{u}}_D}{\sqrt{K}} \right] \quad (8)$$

Either $\beta_{c_\phi} = 0$ or $\Delta C = 0$ results in $N = 0$, or say, only thermal drive applies. Also, for $\beta_{c_\phi} = 0$ and $\Delta C \neq 0$ in Eq. (8), although no concentration drive is modeled, a distribution of C within the field will occur due to the flow established by the thermal drive.

Additional transport equations read (see de Lemos and Tofaneli (2004) for details).

Heat transport

$$(\rho c_p)_f \nabla \cdot (\bar{\mathbf{u}}_D \langle \bar{T} \rangle^i) = \nabla \cdot \{ \mathbf{K}_{eff} \cdot \nabla \langle \bar{T} \rangle^i \} \quad (9)$$

$$\mathbf{K}_{eff} = \underbrace{[\phi \lambda_f + (1-\phi) \lambda_s]}_{\lambda_{eff}} \mathbf{I} + \mathbf{K}_{tor} + \mathbf{K}_t + \mathbf{K}_{disp} + \mathbf{K}_{disp,t} \quad (10)$$

Mass transport

$$\nabla \cdot (\bar{\mathbf{u}}_D \langle \bar{C} \rangle^i) = \nabla \cdot \mathbf{D}_{eff} \cdot \nabla (\phi \langle \bar{C} \rangle^i) \quad (11)$$

$$\mathbf{D}_{eff} = \mathbf{D}_{disp} + \mathbf{D}_{diff} + \mathbf{D}_t + \mathbf{D}_{disp,t} \quad (12)$$

$$\mathbf{D}_{diff} = \langle D \rangle^i \mathbf{I} = \frac{1}{\rho} \frac{\mu_\phi}{Sc} \mathbf{I} \quad (13)$$

$$\mathbf{D}_t + \mathbf{D}_{disp,t} = \frac{1}{\rho} \frac{\mu_{i\phi}}{Sc_t} \mathbf{I} \quad (14)$$

Transport equations for $\langle k \rangle^i$ and its dissipation rate $\langle \varepsilon \rangle^i = \mu \overline{\langle \nabla \mathbf{u}' : (\nabla \mathbf{u}')^T \rangle^i} / \rho$ including additional effects due to temperature and concentration gradients are proposed in as Pedras and de Lemos (2003):

$$\rho \nabla \cdot (\bar{\mathbf{u}}_D \langle k \rangle^i) = \nabla \cdot \left[\left(\mu + \frac{\mu_{i\phi}}{\sigma_k} \right) \nabla (\phi \langle k \rangle^i) \right] + P^i + G^i + G_\beta^i + G_{\beta_C}^i - \rho \phi \langle \varepsilon \rangle^i \quad (15)$$

$$\rho \nabla \cdot (\bar{\mathbf{u}}_D \langle \varepsilon \rangle^i) = \nabla \cdot \left[\left(\mu + \frac{\mu_{i\phi}}{\sigma_\varepsilon} \right) \nabla (\phi \langle \varepsilon \rangle^i) \right] + \frac{\langle \varepsilon \rangle^i}{\langle k \rangle^i} \left[c_1 P^i + c_2 G^i + c_1 c_3 (G_\beta^i + G_{\beta_C}^i) - c_2 \rho \phi \langle \varepsilon \rangle^i \right] \quad (16)$$

where c_1 , c_2 , c_3 and c_k are constants. The generation rate of k due to buoyancy is represented by G_β^i and $G_{\beta_C}^i$ for both the thermal and solute drives, respectively de Lemos and Tofaneli (2004).

2.2. Integral Parameter

The local Nusselt number on the heated inner cylinder for the horizontal cylindrical annuli considering half domain is given by,

$$Nu = -\ln R \left(r \frac{\partial \langle T \rangle^i}{\partial r} \right)_{r=r_i} \quad (17)$$

The average Nusselt number is then given by,

$$\overline{Nu} = -\frac{\ln R}{\pi} \int_0^\pi \left(r \frac{\partial \langle T \rangle^i}{\partial r} \right)_{r=r_i} d\theta \quad (18)$$

3. NUMERICAL DETAILS

The numerical method employed for discretizing the governing equations is the control-volume approach. A hybrid scheme, which includes both the Upwind Differencing Scheme (UDS) and the Central Differencing Scheme (CDS), was used for interpolating the convection fluxes. The well-established SIMPLE algorithm Patankar and Spalding (1972), was followed for handling the pressure-velocity coupling. Individual algebraic equations sets were solved by the SIP procedure of Stone (1968). In addition, concentration of nodal points closer to the walls reduces eventual errors due to numerical diffusion which, in turn, are further annihilated due to the hybrid scheme here adopted. Calculations for laminar and turbulent flows used a 50×50 stretched grid for all cases (Fig. 1b). For turbulent flow calculations, wall log laws were applied.

4. RESULTS AND CONCLUSIONS

The problem considered is showed schematically in Fig. 1 and refers to a concentric annulus completely filled with porous material with outer and inner radii r_o and r_i , respectively, and $R = r_o/r_i = 2$. The cavity is isothermally heated from the inner cylinder and cooled from outer cylinder, with $T_1 > T_2$ and $C_1 > C_2$. The Rayleigh number is defined as $Ra_m = g\beta_\phi(\rho c_p)_f \Delta T K r_i / k_{eff} \nu_f$. As in the case of a square cavity filled with porous material, the parameters (Prandtl number, inertia parameter, conductivity ratio) are fixed.

Figure 2 and 3 shows the isotherms and streamlines of a concentric annuli heated from the inner cylinder and cooled from outer cylinder completely filled with porous material for $Ra_m = 25$, $Ra_m = 200$ and $R = 2$, for buoyancy ratio, $N = 0$. The figure show a good agreement with the work of Braga and de Lemos (2003) and reproduce the basic features of the flow.

Figure 4 shows corresponding isolines of turbulent kinetic energy for $Ra_m = 200$ and $R = 2$. The figure clearly shows that in the upper part of the annular region the turbulent kinetic energy presents its highest levels. The same Fig. 6 when the ratio of thrust is $N = 1$, we can observe that the results to $N = 0$ have a good agreement with the findings in the literature.

Table 1 finally shows, for selected Rayleigh numbers, the average Nusselt number \overline{Nu} based on the heated inner cylinder. In comparison, the turbulent average Nusselt numbers are significantly greater than the ones obtained with a laminar model. A possible explanation for it is that the thin thermal boundary layer above the inner cylinder entails a steeper temperature gradient when turbulence is considered, increasing then the value of the average Nusselt number based on the inner cylinder, the same situation presents to the values of Nu when $N = 1$. Again, it is seen from Tab.1 that the agreement between the present and previous results is reasonable.

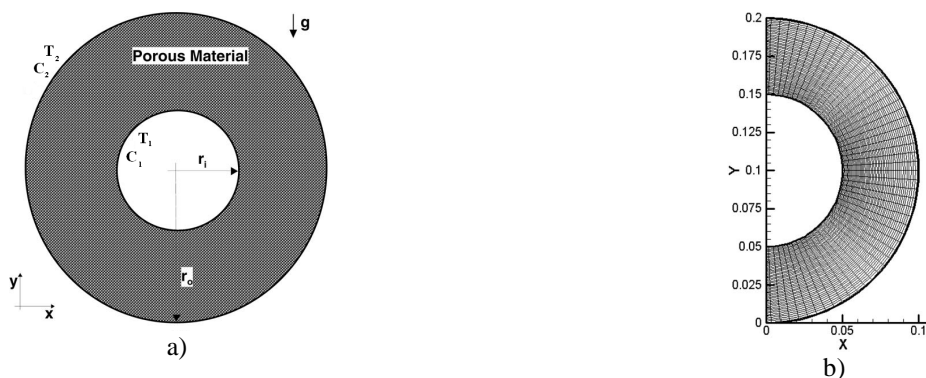


Figure 1: Schematic of the problem: a) geometry; b) grid.

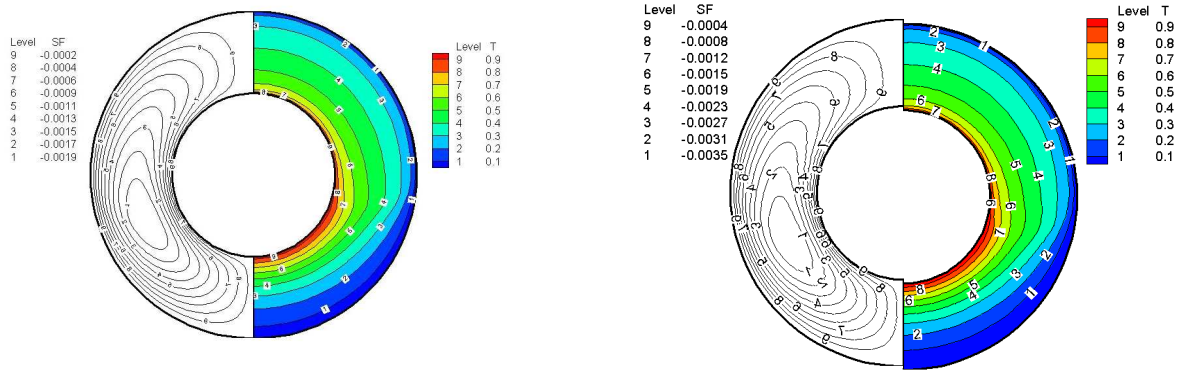


Figure 2: Turbulent Isotherms and Streamlines for $R = 2$, $Ra_m = 25$ with $\phi = 0.2$ and $D_p = 3mm$: a) Presents Results, with $N = 0$, b) Braga and de Lemos (2003) (only thermal model).

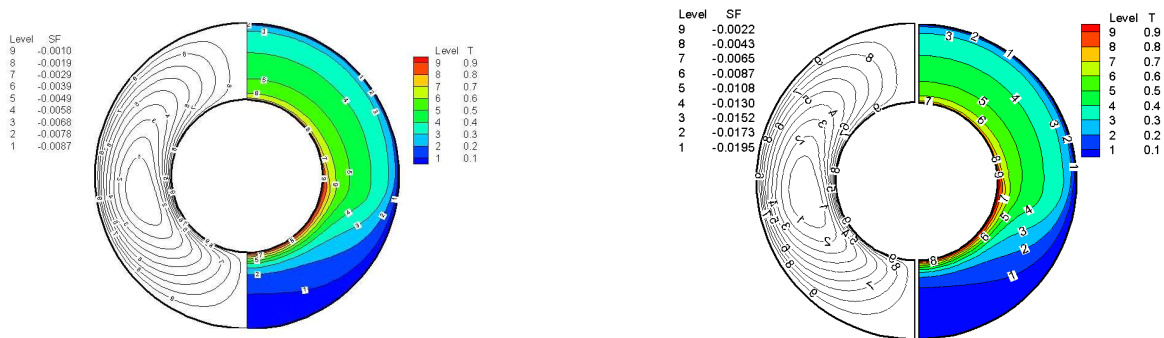


Figure 3: Turbulent Isotherms and Streamlines for $R = 2$, $Ra_m = 200$ with $\phi = 0.2$ e $D_p = 3mm$: a) Presents Results, with $N = 0$, b) Braga and de Lemos (2003) (only thermal model).

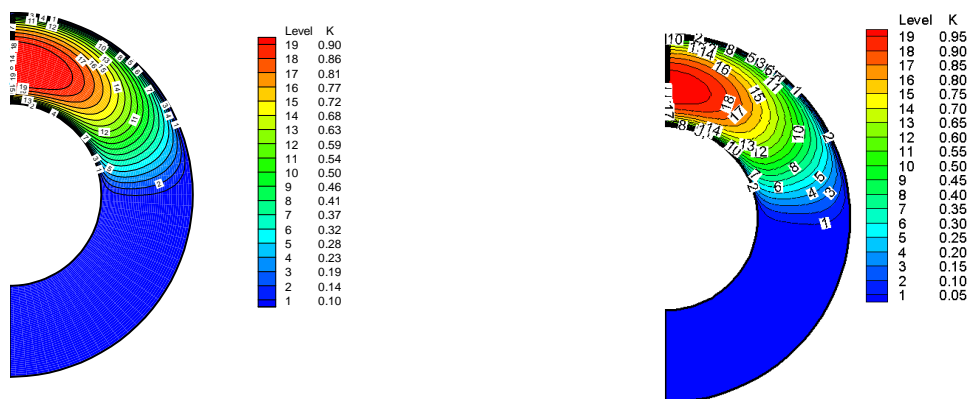


Figure 4: Isolines of turbulent kinetic energy for $R = 2$, $Ra_m = 200$ with $\phi = 0.2$ e $D_p = 3mm$: a) Present Results, with $N = 0$, b) Braga and de Lemos (2003). (only thermal model).

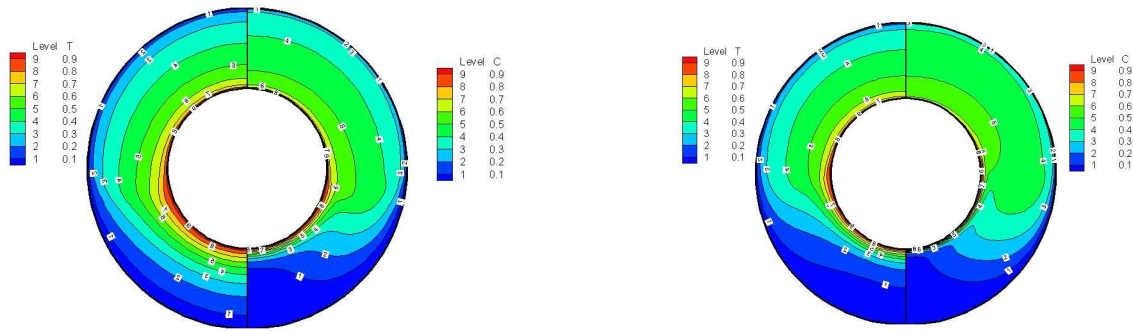


Figure 5: Turbulent Isotherm and Isoconcentration lines for $R = 2$, $\phi = 0.2$ and $D_p = 3mm$: a) $Ra_m = 25$, b) $Ra_m = 200$, with $N = 1$.

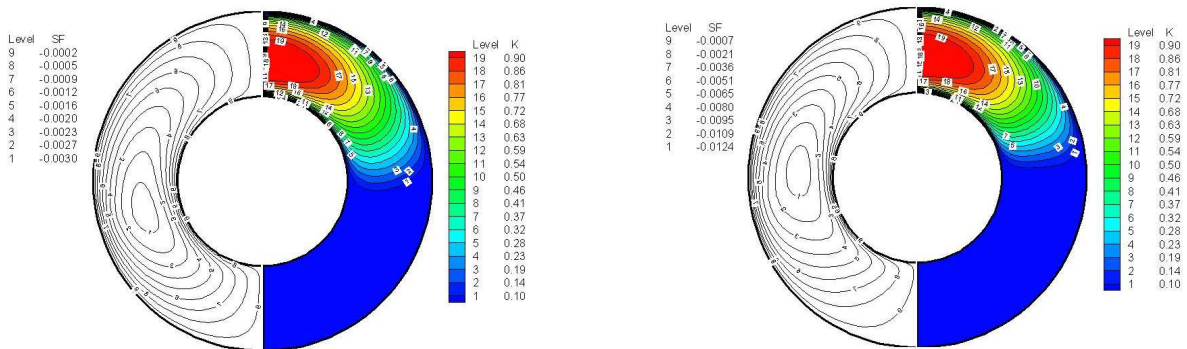


Figure 6: Streamlines and Isolines of turbulent kinetic energy for $R = 2$, $\phi = 0.2$ e $D_p = 3mm$: a) $Ra_m = 25$, b) $Ra_m = 200$ with $N = 1$.

Table 1: Average Nusselt Number for Ra_m ranging from 25 to 500 with, $D_p = 3mm$ and $\phi = 0.2$.

Applied Model \ Ra		25	100	200	500
Laminar Solution	Braga (2003)	1.860	2.296	2.666	4.231
	Presents Results, with $N = 0$	1.856	2.272	2.649	4,263
Turbulent Solution	Braga (2003)	4.689	6.852	7.984	9.450
	Presents Results, with $N = 0$	4.758	6.918	7.626	9.736
With $N = 1$					
Laminar Solution		1.919	2.367	3.198	4.941
Turbulent Solution		4.842	7.025	8.233	9.861

5. ACKNOWLEDGEMENTS

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