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Quantifying cross-correlation between Ibovespa and Brazilian blue-chips: The DCCA approach

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HIGHLIGHTS

- We studied the cross-correlation in function of time between Brazilian blue-chips and Ibovespa.
- We applied the ρ_{DCCA} coefficient to measure these cross-correlation.
- We also analyzed the 2008 financial crisis in terms of ρ_{DCCA}.

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ABSTRACT

The objective of this paper is to demonstrate the influence of the blue-chips companies in the stock market. In this, we apply the detrended cross-correlation coefficient ρ_{DCCA} at the São Paulo stock market (Ibovespa, Brazil). Initially we found that there is a positive cross-correlation between these companies and the index. Afterwards, we show that the cross-correlation coefficient value depends on the time scale and the specific company (blue-chips). Thus, this type of analysis lets to infer what is the most adherent company with Ibovespa. Also, in this paper we analyze, in the point of view of ρ_{DCCA}, the 2008 financial crisis (before/after). Altogether, the results show that there is more cross-correlation between Ibovespa index the blue-chips after the 2008 crisis.

1. Introduction

In financial markets, the blue-chips are the top stocks, which have high liquidity, high reliability and a good reputation. These are usually instruments of companies that are essential for the development of a country or a strategic sectors. In Brazil, they are mostly from commodity or financial sector. In this case, Ibovespa is the main index applied to make decisions, because it is a gross total return index weighted by traded volume and is comprised of the most liquid stocks traded on São Paulo stock exchange. Ibovespa is composed of 68 companies, with their goal being the average index of the prices of

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assets with greater marketability and representativeness of the Brazilian economy [1]. Thus, from the perspective of the nine largest companies that compose the index (the blue-chips), this paper aims to identify and measure the cross-correlation between these blue-chips and Ibovespa. In this sense, we apply the detrended cross-correlation coefficient ρ_{DCCA} [2]. We know that ρ_{DCCA} was established in order to quantify the level of cross-correlation between two non-stationary time series. The ρ_{DCCA} coefficient is defined as the ratio between the detrended covariance function (by DCCA method [3]) and the detrended variance function (by DFA method [4]), and can be considered as an evolution of the DCCA method, widely used in recent years [5–9]. The ρ_{DCCA} was applied in meteorology [10], in time series of homicide and attempted homicide [11], in economy [12–17], among others. Furthermore, the ρ_{DCCA} was compared with Pearson's correlation coefficient and has been shown more efficient [18], mainly for two non-stationary time series. Recently, Kristoufek [19] developed another method of cross-correlation analysis based on the moving average cross-correlation coefficient, to make the cross-correlation more robust.

In this paper we intend to apply the ρ_{DCCA} to investigate the cross-correlations between Brazilian blue-chips companies and Ibovespa. Certainly the study of such relationships are important in finance, and the connections between stocks and stock indices are frequently used in trading strategies and portfolio constructions (mainstream financial economics around theories of Markowitz, Sharpe, CAPM, etc.). Therefore, this paper is organized as follows: in Section 2 we present the methodology, in Section 3 we describe the data and the results, and in Section 4 we make the conclusions.

2. Methodology

2.1. The ρ_{DCCA} coefficient [2]

The detrended cross-correlation coefficient is a new method to quantify the level of cross-correlation between two non-stationary time series. This method is based on detrended fluctuation analysis (DFA) [4], and detrended cross-correlation analysis (DCCA) [3]. Thus, for a better understanding of ρ_{DCCA} below we present the algorithm in four steps:

Step I: considering two time series, $\{x_t\}$ and $\{y_t\}$, with $t = 1, 2, \ldots, N$, (N is the time series length). Then we integrated these time series, obtaining two new series

$$xx_k = \sum_{t=1}^{k} x_t \quad \text{and} \quad yy_k = \sum_{t=1}^{k} y_t, \quad k = 1, 2, \ldots, N. \tag{1}$$

Step II: we divide these two integrated time series, $\{xx_k\}$ and $\{yy_k\}$, into $(N - s)$ overlapping boxes of equal length s, with $4 \le s \le N/4$.

Step III: we calculate the local trend of each box by a least-squares fit of each series, $xP_i\,(k)$ and $yP_i\,(k)$. Next, we calculate the covariance of the residuals in each box by:

$$F^2_{xy}\,(s, i) = \frac{1}{s+1} \sum_{k=i}^{i+s} (xx_k - xP_i\,(k))\,(yy_k - yP_i(k))\,. \tag{2}$$

Step IV: the average over all $(N - s)$ overlapping boxes is calculated to obtain the new covariance function:

$$F^2_{xy}\,(s) = \frac{1}{(N-s)} \sum_{i=1}^{N-s} F^2_{xy}\,(s, i)\,. \tag{3}$$

Step V: finally we calculate the cross-correlation coefficient ρ_{DCCA} by:

$$\rho_{\mathrm{DCCA}}(s) = \frac{F^2_{xy}\,(s)}{F_{xx}\,(s)\,F_{yy}(s)}. \tag{4}$$

This cross-correlation coefficient, as we can see, depends on the box length s (time scale). One of the advantages of this cross-correlation coefficient is that it measures the correlations between two non-stationary time series at different time scales. The DCCA cross-correlation coefficient ranges from $-1 \le \rho_{\mathrm{DCCA}} \le 1$, logically 1 means perfectly cross-correlation, -1 means perfectly anti-cross-correlation, and 0 means there is no cross-correlation. For more details see Ref. [20].

Taking into account that has been said above, in this paper we analyze the cross-correlation between Ibovespa and the blue-chips from a well-defined relationship between α_{DFA} and λ_{DCCA} exponents [21]. Thus, in the following section we present briefly this theory in the sense of improving this paper.

2.2. Differential of ρ_{DCCA} coefficient

According to Peng et al. [4], the power law that describes the self-correlation of two non-stationary series with power law can be written as:

$$F_{xx}(s) = K_{xx}s^{\alpha_1}, \qquad F_{yy}(s) = K_{yy}s^{\alpha_2} \tag{5}$$

and according to Ref. [3] for power law cross-correlation we have,

$$F^2_{xy}(s) = K_{xy}s^{2\lambda} \tag{6}$$

Table 1
Ibovespa blue-chips.

ID	Blue-chips
bbas3	Bank of Brazil
bbdc3	Bradesco Bank
csna3	National Steel
ggbr4	Gerdau Group
itsa4	Itaú Bank
itub4	Itaú-Unibanco
petr4	Petrobrás
usim5	Usiminas
vale3	Vale do Rio Doce

here K_{xx}, K_{yy} and K_{xy} are positive constants. From Eqs. (4), (5), and (6)

$$\rho_{\mathrm{DCCA}}(s) = Ks^{2\lambda-\alpha_1-\alpha_2}, \tag{7}$$

with $K \equiv \frac{K_{xy}}{K_{xx}K_{yy}}$.

If we define $y \equiv \log_{10} \rho_{\mathrm{DCCA}}(s)$ and $x \equiv \log_{10}(s)$, then the derivative

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\lambda - \alpha_1 - \alpha_2. \tag{8}$$

Thus, a relationship between α_1, α_2 and λ can be given by

$$\lambda = \frac{(\mathrm{d}y/\mathrm{d}x + \alpha_1 + \alpha_2)}{2}. \tag{9}$$

According to Ref. [20], $\mathrm{d}y/\mathrm{d}x$ can help us understand better the cross-correlations in non-stationary time series.

3. Data and results

We analyze the average daily of stock prices in the financial and commodities sectors of São Paulo (Brazil) stock market. These data are from Ibovespa and nine blue-chips companies, that compose this index (see Table 1).

These data were collected daily between October 22, 2001 and October 22, 2014 from Ref. [22]. These nine blue-chips were the most traded in 2011 [23], because they reached US$ 1.6 billion per day, which represented 47.2% of the total market. Vale PNA[1] achieved the highest average annual daily trading volume in 2011, with US$ 416.2 million per day on average. In this paper we consider 2011 as the base year for the blue-chips formation. Now, considering $P_i(t)$ as the share price of company i on day t, the return of company i, is defined by,

$$r_i(t) \equiv \log_{10} \frac{P_i(t)}{P_i(t-1)}. \tag{10}$$

For our understanding and data visualization Fig. 1 shows the return of Ibovespa for all periods (a), before the 2008 crisis (b), and after the 2008 crisis (c).

Table 2 shows the descriptive statistical of the returns for Ibovespa and the blue-chips, for the entire time series, before, and after the 2008 crisis.

The statistical descriptive for the entire time series shows a negative skewness, except for *bbdc3* and *itsa4* that show a positive value. For Ibovespa, the skewness was negative for entire and pre-crisis, but in the post-crisis the skewness was positive with value approximately zero. The percentile coefficient of kurtosis in Table 2 is defined by:

$$Kurtosis \equiv \frac{Q_3 - Q_1}{2(P_{90} - P_{10})}, \tag{11}$$

where Q_i is the ith quartile of the returns and P_i is the order i percentile. In this way, a normal distribution has $Kurtosis = 0.263$. In Table 2, every value of *Kurtosis* is less than 0.263, denoting returns with behaviors less acute than normal distribution (long tail).

Taken a descriptive analysis of the series, henceforward we apply the ρ_{DCCA}. The results are found in Fig. 2. This figure shows the detrended cross-correlation coefficient value between Ibovespa and all nine blue-chips. Here we present the

[1] The "PN", from Portuguese Preferência Normativa, means an action that entitles its holder priority to receive dividends and/or, in the case of dissolution of the company, the repayment of capital stock. In general, it does not provide direct voting in assembly. Actions can also be differentiated by classes: A, B, C or some other letter that appears after the "PN". The characteristics of each class are set by the company issuing the share in its bylaws. These differences vary from company to company, so it is not possible to make a general definition of the classes of shares. For more information: http://folhainvest.folha.com.br/faq#pn_on_pnb_pna.

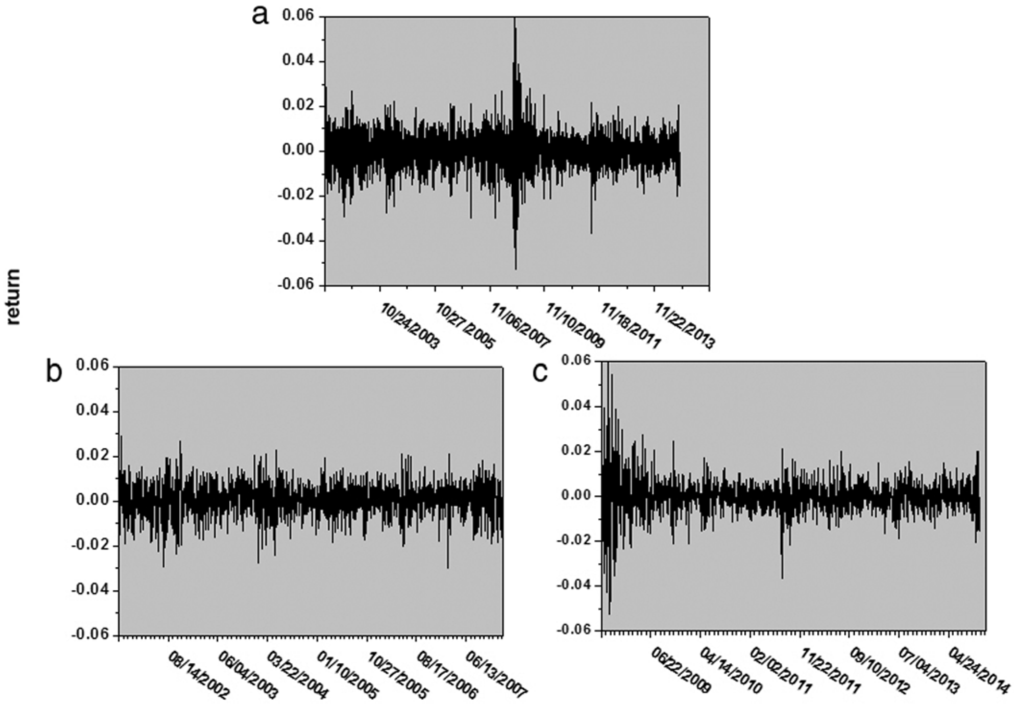

Fig. 1. Return of Ibovespa for: (a) for data collected between October 22, 2001 and October 22, 2014, (b) before the 2008 crisis, and (c) post 2008 crisis.

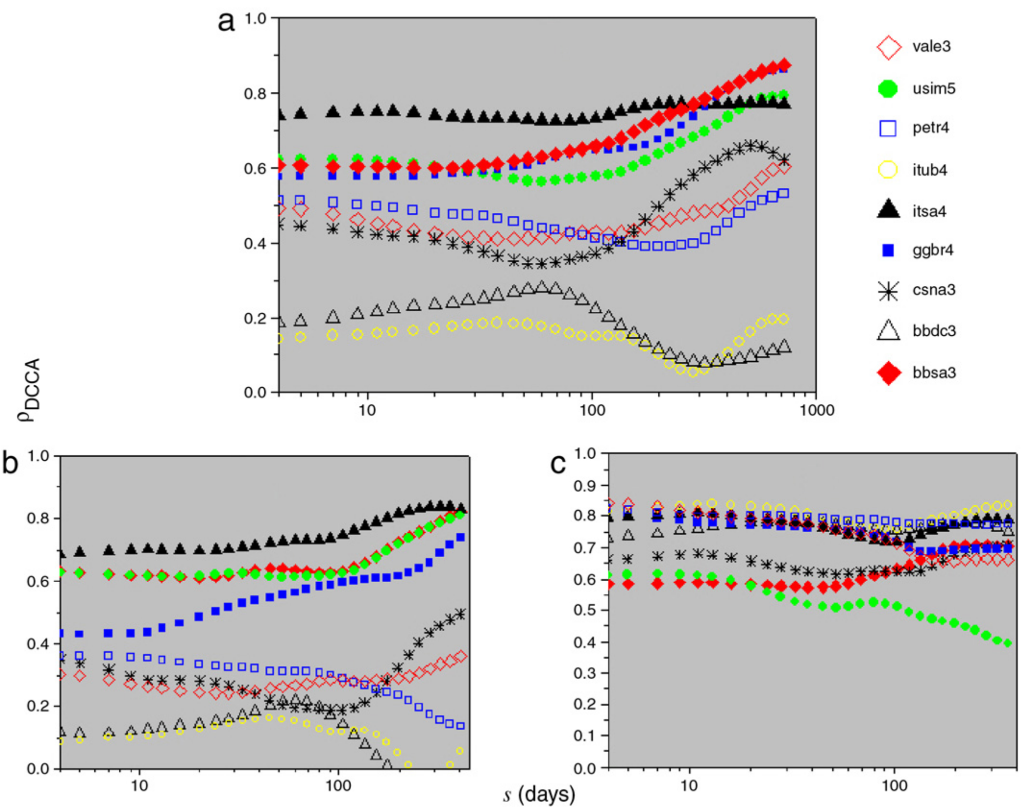

Fig. 2. (Color online) Detrended cross-correlation coefficient between Ibovespa and the nine blue-chips companies in function of time scale *s*. (a) Represents the entire time series analysis, (b) the 2008 pre-crisis, and (c) the 2008 post-crisis analysis.

Table 2
(Color on-line) Statistical description of the returns of the prices of the nine blue-chips and Ibovespa Index for: (i) entire time series (left cell), (ii) before the 2008 crisis (top right), and (iii) after the 2008 crisis (bottom right).

	bbas3		*bbdc3*		*csna3*		*ggbr4*		*itsa4*		*itub4*		*petr4*		*usim5*		*vale3*		Ibovespa	
Mean	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000	-0.001	-0.001	0.000	0.000	0.000	0.001	0.000	0.000	**0.000**	**0.000**
		0.000		0.000		-0.001		0.000		0.000		0.000		0.000		-0.001		0.000		**0.000**
Maximum	0.114	0.062	0.994	0.994	0.470	0.470	0.210	0.210	0.097	0.033	0.091	0.042	0.058	0.058	0.082	0.043	0.059	0.048	**0.059**	**0.029**
		0.082		0.067		0.085		0.073		0.097		0.091		0.053		0.082		0.059		**0.059**
Minimum	-0.296	-0.039	-1.002	-1.001	-0.596	-0.596	-0.307	-0.307	-0.066	-0.058	-2.995	-2.995	-0.602	-0.602	0.290	0.152	-0.469	-0.469	**-0.053**	**-0.030**
		-0.296		-0.055		-0.294		-0.070		-0.066		0.056		-0.064		-0.290		-0.089		**-0.053**
sd	0.013	0.012	0.032	0.046	0.020	0.025	0.015	0.017	0.010	0.009	0.053	0.074	0.015	0.019	0.015	0.014	0.015	0.018	**0.008**	**0.008**
		0.014		0.001		0.015		0.012		0.011		0.011		0.011		0.016		0.011		**0.008**
Skewness	-3.166	0.328	8.087	5.960	-8.581	-8.516	-5.091	-6.916	0.135	-0.141	-54.813	-39.625	-19.734	-21.976	-2.760	-1.459	-14.421	-15.558	**-0.104**	**-0.240**
		-6.814		0.017		-4.678		0.064		0.318		0.496		-0.092		-3.992		0.132		**0.075**
Kurtosis	0.244	0.260	0.230	0.240	0.241	0.260	0.247	0.257	0.236	0.246	0.243	0.249	0.249	0.257	0.243	0.260	0.217	0.242	**0.245**	**0.251**
		0.245		0.245		0.235		0.243		0.223		0.231		0.243		0.233		0.232		**0.234**
***N* (obs)**	3547	1656	3532	1655	3557	1656	3332	1653	3408	1653	3372	1651	3354	1652	3386	1653	3552	1656	**3409**	**1656**
		1508		1516		1508		1455		1507		1497		1477		1495		1510		**1524**

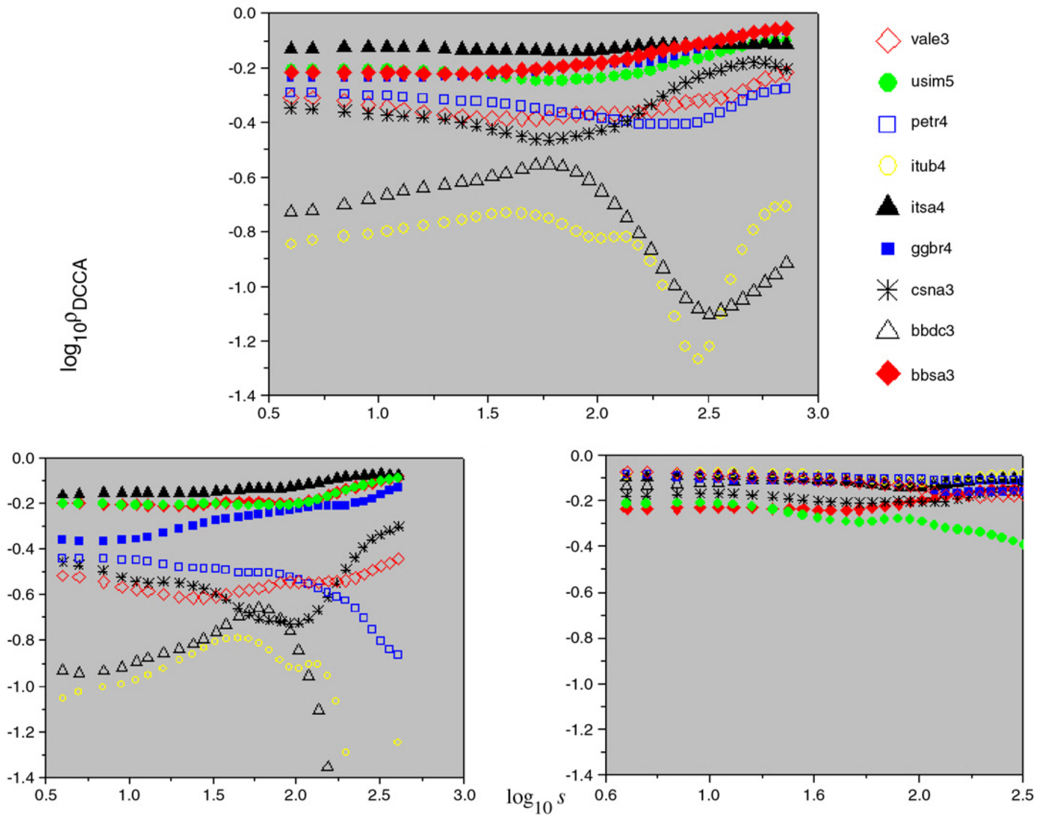

Fig. 3. (Color online) The $y = \log_{10} \rho_{\mathrm{DCCA}}$ in function of $x = \log_{10} s$ for cross-correlation between Ibovespa and the blue-chips: (a) entire time series analysis, (b) before the 2008 crisis, and (c) after the 2008 crisis.

complete time series analysis (Fig. 2(a)), before the 2008 crisis (Fig. 2(b)), and after the 2008 crisis (Fig. 2(c)). The results show that ρ_{DCCA} is always positive, and there is a clear adhesion between the blue-chips and Ibovespa. But, even more, we can see an adherence scale, for example:

(a) ρ_{DCCA} (*itsa*4) $>$ ρ_{DCCA} (*itub*4) for any time scale;
(b) ρ_{DCCA} (*petr*4) $>$ ρ_{DCCA} (*csan*3) for "short" time scale $<$100 days;
(c) ρ_{DCCA} (*petr*4) $<$ ρ_{DCCA} (*csan*3) for "long" time scale $>$200 days.

In our results we implement the 2008 crisis (Fig. 2(b) and (c)). These results show an interesting behavior. If only the pre-crisis period is analyzed (Fig. 2(b)), then we can see similar qualitative behaviors with Fig. 2(a). However, the post-crisis period (Fig. 2(c)) shows more adherent behavior between the blue-chips and Ibovespa. To observe the ρ_{DCCA} variation, we apply the differential test [20]. The results are found in Fig. 3. This type of analysis corroborates with Fig. 2 as expected, but with more details. In general *itsa4*, *usim5*, *ggbr4*, and *bbsa3* had $\mathrm{d}y/\mathrm{d}x \approx 0$, for every cases (entire, before, and after the 2008 crisis). While for the remaining companies this has not occurred. In the pre-crisis such a behavior is more evident, because *itub4*, *bbdc3*, *csna3*, *vale3*, and *petr4* fluctuated with time scale. Surprisingly, in the post-crisis we can see $dy/dx \approx 0$ for all blue-chips, showing almost perfect data adherence for Ibovespa.

4. Conclusions

In general terms we found a positive cross-correlation between Ibovespa and the blue-chips, but depending on the specific company, this cross-correlation can be more or less evident. If we analyze the cross-correlation in the point of view of the 2008 financial crisis, we can see more adherence between Ibovespa and the blue-chips in the post-crisis period. This becomes even more pronounced in the point of view of dy/dx (Fig. 3). This distinct behavior between before and after 2008 crisis, can be a consequence of economic policies implemented during the financial crisis by the Brazilian government. Finally, this paper implements a new statistical analysis of financial adherence, and for this reason is an important way for governments and companies to make decisions.

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